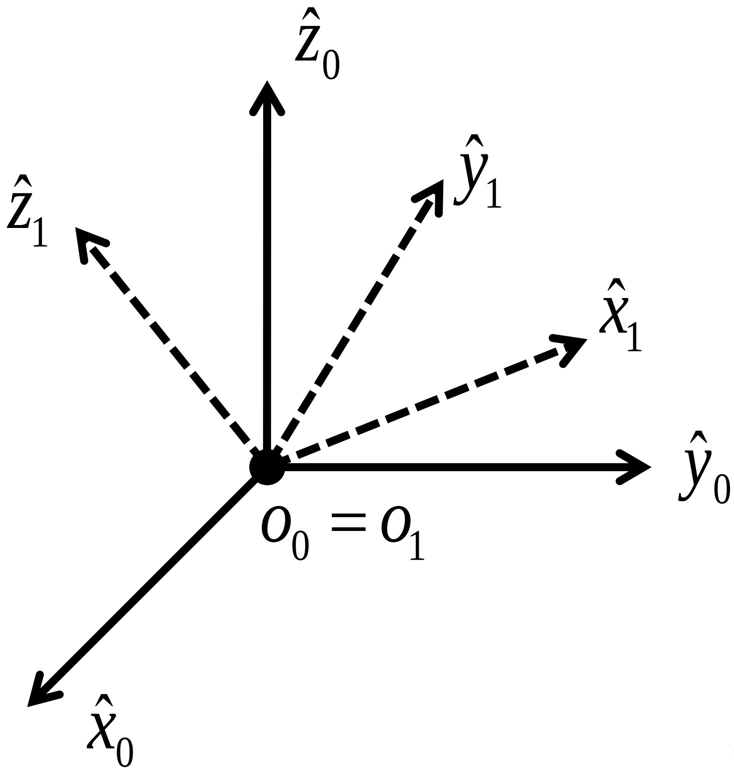
Day 03

Rotations

Properties of Rotation Matrices

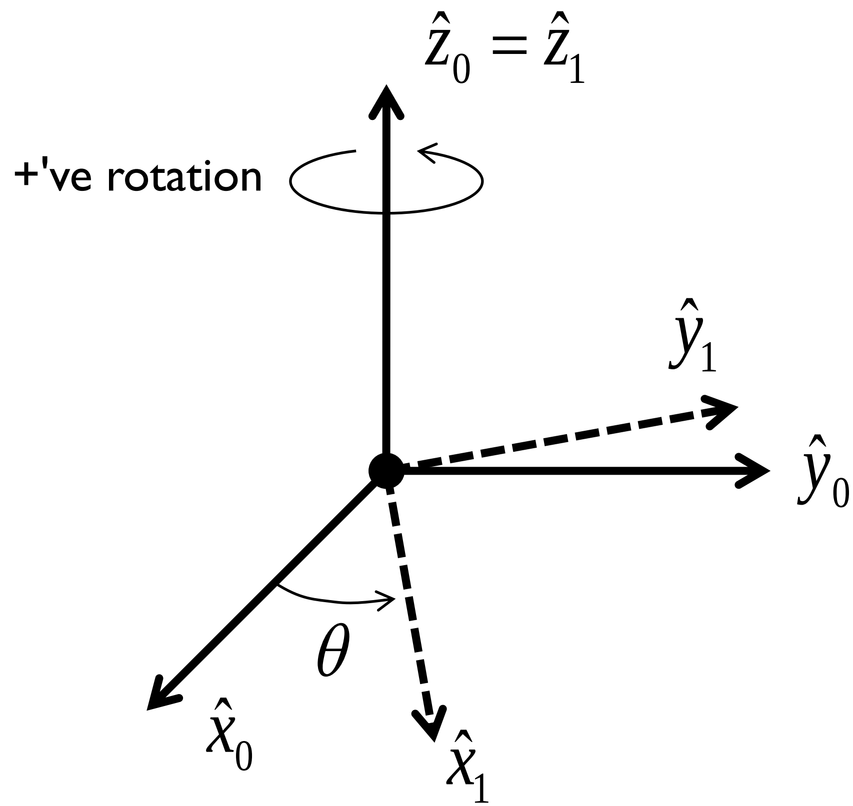
- $R^T = R^{-1}$
- the columns of R are mutually orthogonal
- each column of R is a unit vector
- $\det R = 1$ (the determinant is equal to 1)

Rotations in 3D

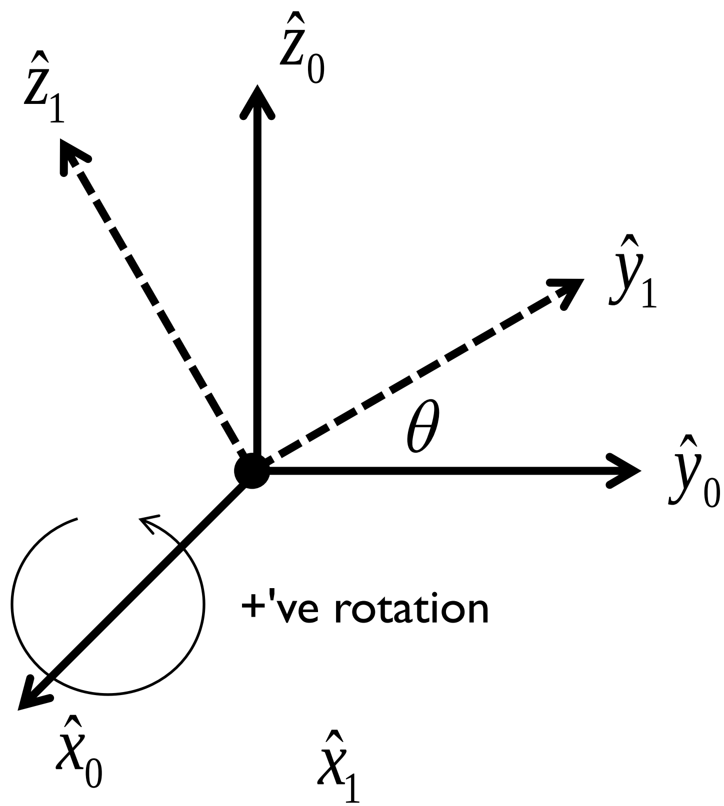


$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

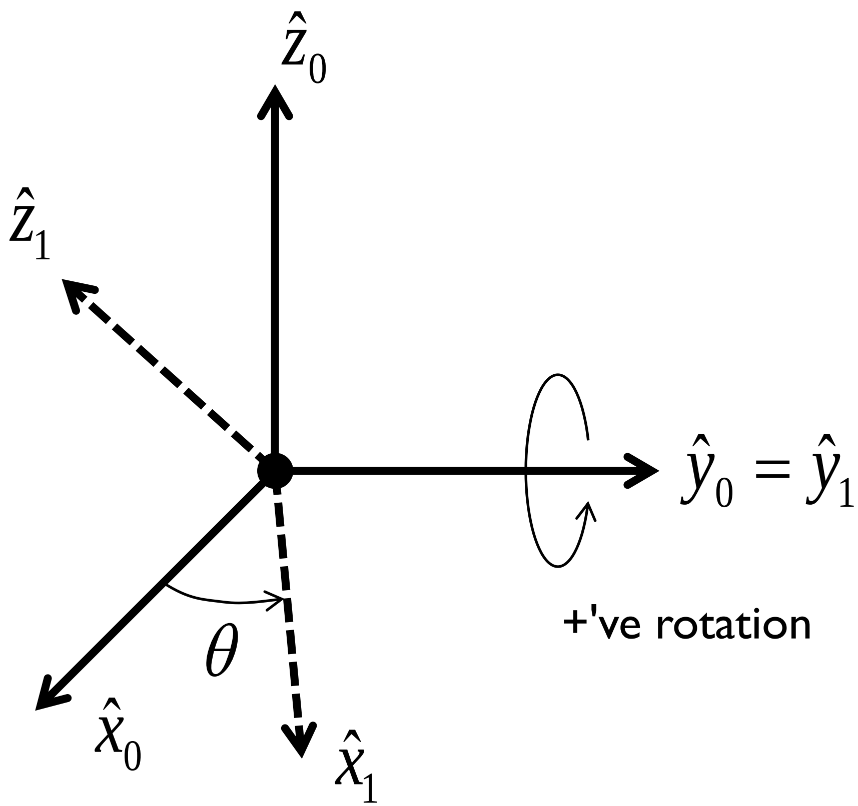
Rotation About z-axis



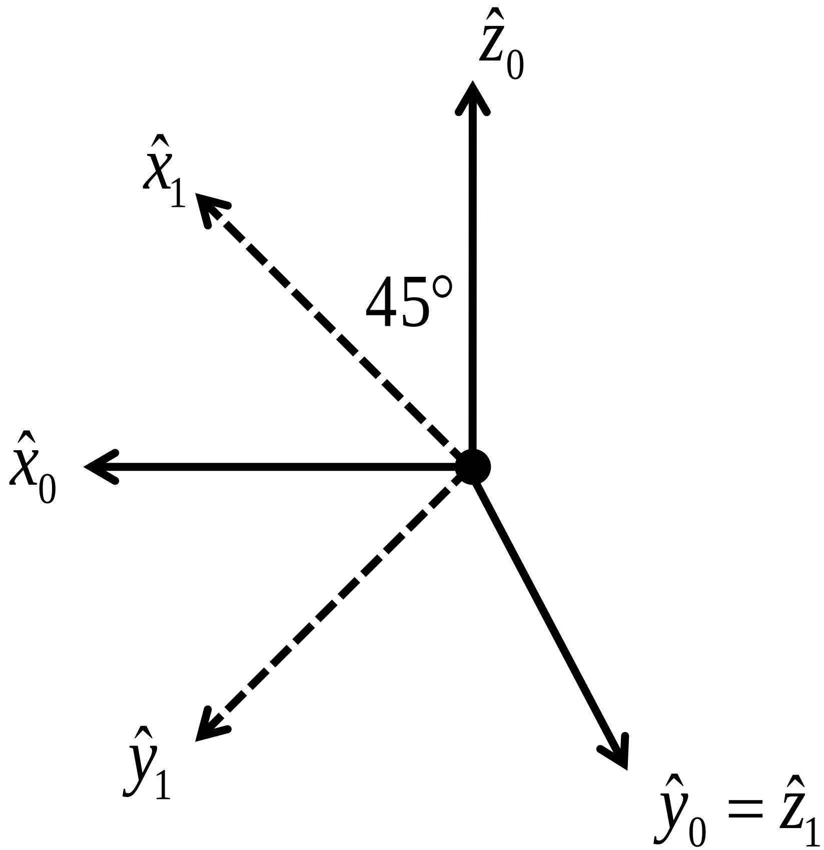
Rotation About x-axis



Rotation About y-axis



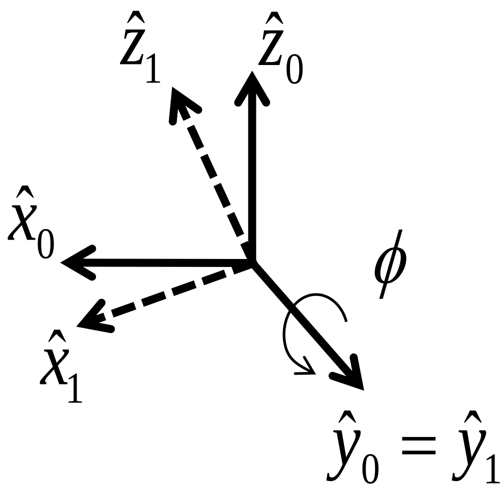
Relative Orientation Example



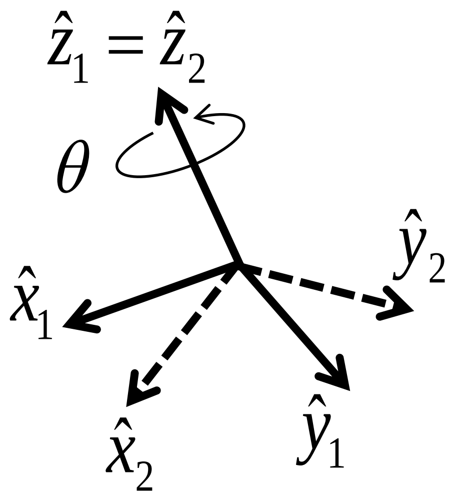
Successive Rotations in Moving Frames

1. Suppose you perform a rotation in frame {0} to obtain {1}.
2. Then you perform a rotation in frame {1} to obtain {2}.

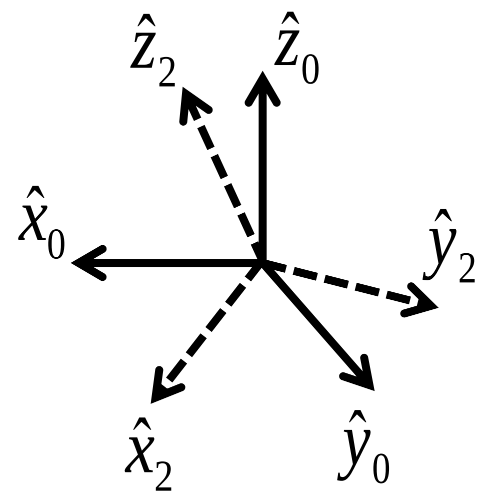
What is the orientation of {2} relative to {0}?



$R_1^0 = R_{y,\phi}$



$R_2^1 = R_{z,\theta}$

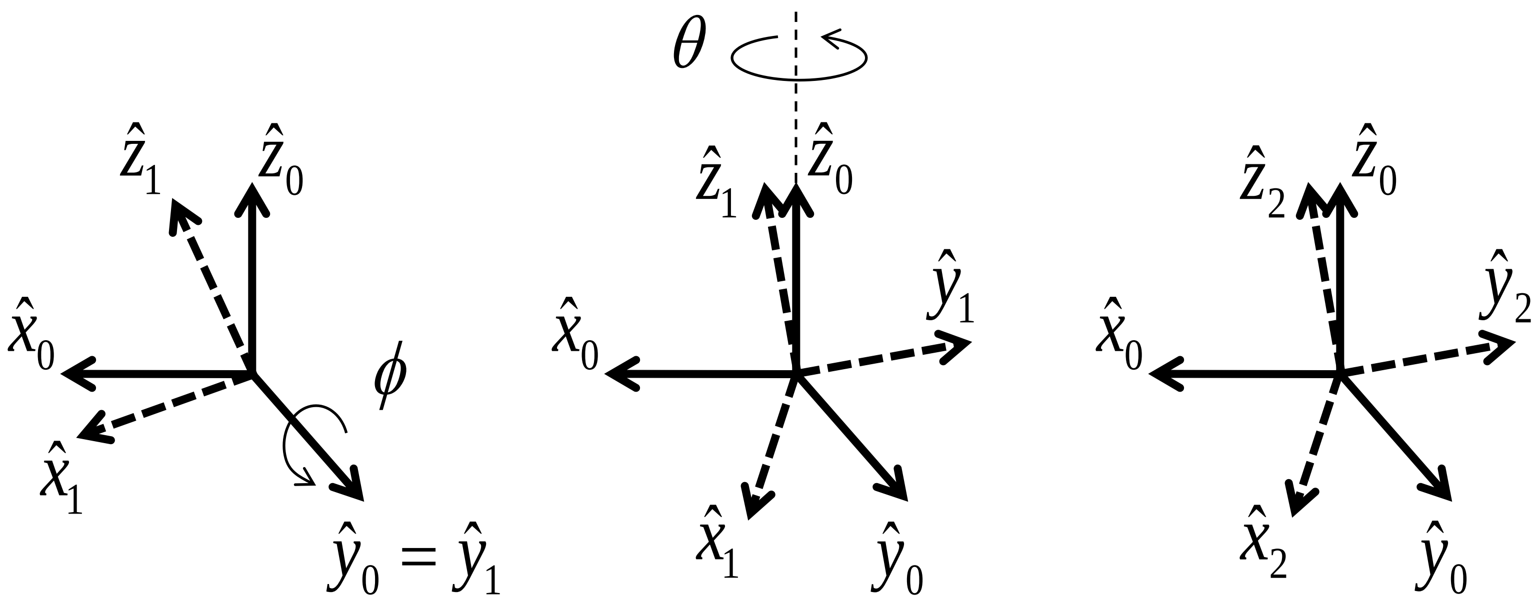


$R_2^0 = ?$

Successive Rotations in a Fixed Frame

1. Suppose you perform a rotation in frame {0} to obtain {1}.
2. Then you rotate {1} in frame {0} to obtain {2}.

What is the orientation of {2} relative to {0}?



$$R_1^0 = R_{y,\phi} \qquad R = R_{z,\theta} \qquad R_2^0 = ?$$

Composition of Rotations

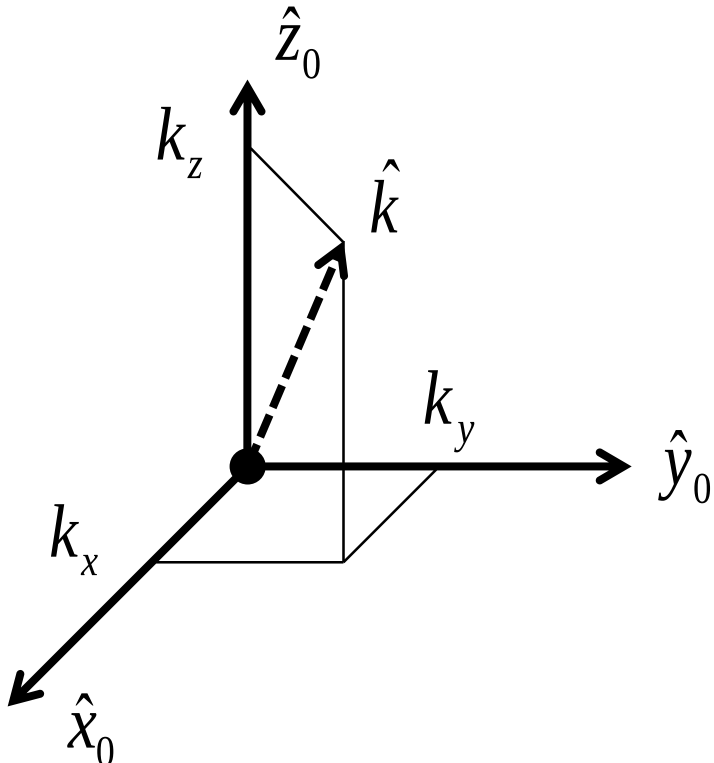
1. Given a fixed frame {0} and a current frame {1} and R_1^0
   - if {2} is obtained by a rotation R in the *current frame* {1} then use postmulitplication to obtain:

$$R = R_2^1 \quad \text{and} \quad R_2^0 = R_1^0 R_2^1$$

2. Given a fixed frame {0} and a frame {1} and
   - if {2} is obtained by a rotation R in the *fixed frame* {0} then use premultiplication to obtain:

$$R_2^0 = R R_1^0$$

Rotation About a Unit Axis



$$c_\theta = \cos\theta$$

$$s_\theta = \sin\theta$$

$$v_\theta = 1 - \cos\theta$$

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$